SEKCJA 21. FIZYKI I MATEMATYKI.
(ФИЗИКО-МАТЕМАТИЧЕСКИЕ НАУКИ)

Subsection 6. Mathematics.

Chubatov A. A., Karmazin V. N.
Chubatov Andrey Alexeevich, senior teacher,
Dept. of mathematics of Armavir Pedagogical State Academy, Armavir, Russia
Karmazin Vladimir Nikolaevich, Ph.D. (Phis. & Math.) Prof.,
Dept. of applied mathematics of Kuban State University, Krasnodar, Russia

THE EXPRESS MONITORING OVER ATMOSPHERIC POLLUTION SOURCE ON THE BASE OF FUNCTION SPECIFICATION METHOD

Abstracts. The approach allows to sequential estimate action intensity of atmospheric pollution source on the base of impurity concentration measurements in several stationary control points is offered in the work. The inverse problem was solved by means of the function specification method. The solution is presented in the form of the digital filter.

Key words: *inverse problem, ill-posed problem, function specification method, on-line monitoring, atmospheric pollution, turbulent diffusion equation.*

Introduction. For the description of processes of impurity distribution in the atmosphere (domain $D\times[0;H]$) it is used three-dimensional linear turbulent diffusion equation [1]

$$\partial q/\partial t + v\cdot \text{grad } q = \text{div}(K\cdot \text{grad } q) + f(x, y, z)\cdot g(t), \quad (1)$$

at following conditions $q\big|_{t=0} = h(x,y,z)$, $q\big|_{\partial D\times[0;H]} = 0$.

where $q(x,y,z,t)$ — concentration of an pollution impurity, (v_x, v_y, v_z) — vector of speeds of a wind, (K_x, K_y, K_z) — vector of coefficients of turbulent diffusion, $f(x,y,z)$ — function describing spatial arrangement of a pollution source, $g(t)$ — action intensity of source.

The inverse problem of intensity identification of pollution impurity emissions consists in sequential estimation of function $g(t)$ according to concentration measurements in stationary control points, located in points (x_j, y_j, z_j), $j = 1, 2, \ldots, J$. Measurements are taken in time intervals Δt.

Let's consider, that an error of concentration measurements is additive

$$c_{ji} = q(x_j, y_j, z_j, t_i) + \delta \cdot \gamma ,$$

where c_{ji} — concentration measured by j th sensor at the moment of time $t_i = i \cdot \Delta t$, δ — root-mean-square error of sensor measurements, — standardized Gaussian random variable (Average(γ)=0, Variance(γ)=1).

Let $g(t)$ accepts each time interval $[t_{N-1}; t_N]$ constant value g_N .

The inverse problem for a source is characterized by solution instability to errors of concentration measurements also demands special methods of the solution [2,3,4]. To solve the problem were used methods step-by-step regularization and sequential function specification [3,4].

Sequential function specification method. Linearity of the problem (1) allows to use the superposition principle and numerical analogue of Duhamel's theorem

$$q(x_j, y_j, z_j, t_i) = Q_{init}(x_j, y_j, z_j, t_i) + \sum_{n=1}^{i} g_n \cdot \left(Q(x_j, y_j, z_j, t_{i-n+1}) - Q(x_j, y_j, z_j, t_{i-n}) \right),$$
(2)

where $Q(x, y, z, t)$ — solution of the direct problem (1) at $g(t) = 1$ and $q\big|_{t=0} = 0$, $Q_{init}(x, y, z, t)$ — solution of the direct problem (1) at $g(t) = 0$ and $q\big|_{t=0} = h(x, y, z)$.

Let's enter designations $q(x_j, y_j, t_i) = q_{ji}$, $Q(x_j, y_j, z_j, t_i) = \phi_{ji}$, $Q_{init}(x_j, y_j, z_j, t_i) = \phi^0_{ji}$, $\phi_{j(i-n+1)} - \phi_{j(i-n)} = \Delta\phi_{j(i-n)}$. The value ϕ_{ji} is called step sensitivity coefficient, and the value $\Delta\phi_{ji}$ — pulse sensitivity coefficient.

We shall estimate g_N , considering $g_1, g_2, \ldots, g_{N-1}$ are known values, calculated on the previous steps. For giving stability to the solution of the inverse problem we shall consider $g(t)$ on several (r) time intervals at once. Let's consider, that $g_N, g_{N+1}, \ldots, g_{N+r-1}$ are connected by some functional dependence. At $r = 1$ the method step-by-step regularization turns out.

Using (2) for the moments of time $t_N, t_{N+1}, \ldots, t_{N+r-1}$ let's write down the matrix equation

$$\ddot{\mathbf{Q}} = \mathbf{Q}_{init} + \mathbf{Q}\big|_{\mathbf{g=0}} + \quad \cdot \quad , \qquad (3)$$

where $\mathbf{Q}, \mathbf{Q}\big|_{\mathbf{g=0}} \in \mathbb{R}^{r\cdot J}$, $\boldsymbol{f} \in \mathbb{R}^{r\cdot J\times r}$, $\mathbf{G} \in \mathbb{R}^{r}$, $\mathbf{Q_{N+k}}, \mathbf{Q_{N+k}}\big|_{\mathbf{g=0}} \in \mathbb{R}^{J}$, $\ddot{\mathbf{O}}_{\mathbf{k}} \in \mathbb{R}^{J}$,

$k = 0 \div r-1$,

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q_N} \\ \cdots \\ \mathbf{Q_{N+r-1}} \end{bmatrix}, \ \mathbf{Q_{N+k}} = \begin{bmatrix} q_{1(N+k)} \\ \cdots \\ q_{J(N+k)} \end{bmatrix}, \ \mathbf{Q}_{init} = \begin{bmatrix} \mathbf{Q}^{init}_{\mathbf{N}} \\ \cdots \\ \mathbf{Q}^{init}_{\mathbf{N+r-1}} \end{bmatrix}, \ \mathbf{Q}^{init}_{\mathbf{N+k}} = \begin{bmatrix} \phi^{0}_{1(N+k)} \\ \cdots \\ \phi^{0}_{J(N+k)} \end{bmatrix},$$

$$\mathbf{Q}\big|_{\mathbf{g=0}} = \begin{bmatrix} \mathbf{Q_N}\big|_{\mathbf{g=0}} \\ \cdots \\ \mathbf{Q_{N+r-1}}\big|_{\mathbf{g=0}} \end{bmatrix}, \ \mathbf{Q_{N+k}}\big|_{\mathbf{g=0}} = \begin{bmatrix} q_{1(N+k)}\big|_{g=0} \\ \cdots \\ q_{J(N+k)}\big|_{g=0} \end{bmatrix}, \ \boldsymbol{f} = \begin{bmatrix} \boldsymbol{f}_{\mathbf{0}} & 0 & 0 & 0 \\ \boldsymbol{f}_{\mathbf{1}} & \boldsymbol{f}_{\mathbf{0}} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{f}_{\mathbf{r-1}} & \boldsymbol{f}_{\mathbf{r-2}} & \cdots & \boldsymbol{f}_{\mathbf{0}} \end{bmatrix},$$

$$\ddot{\mathbf{O}}_{\mathbf{k}} = \begin{bmatrix} \Delta\phi_{1k} \\ \cdots \\ \Delta\phi_{Jk} \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} g_N \\ \cdots \\ g_{N+r-1} \end{bmatrix}.$$

We minimize the sum of squares of differences between measured C and calculated $\mathbf{Q}$ values of concentration

$$S = (\mathbf{C}-\mathbf{Q})^{T} \cdot (\mathbf{C}-\mathbf{Q}) \rightarrow \min_{\mathbf{G}}, \qquad (4)$$

where $\mathbf{C} \in \mathbb{R}^{r\cdot J}$, $\mathbf{C_{N+k}} \in \mathbb{R}^{J}$, $\mathbf{C} = \begin{bmatrix} \mathbf{C_N} \\ \cdots \\ \mathbf{C_{N+r-1}} \end{bmatrix}$, $\mathbf{C_{N+k}} = \begin{bmatrix} c_{1(N+k)} \\ \cdots \\ c_{J(N+k)} \end{bmatrix}$.

Let's temporarily assume, that $g_{N+1}, g_{N+2}, \ldots, g_{N+r-1}$ it is expressed by means of some functional dependence from g_N and g_{N-1}, we shall estimate unknown value g_N and we shall pass to a following step, temporarily assuming dependence

$g_{(N+1)+1}, g_{(N+1)+2}, \dots, g_{(N+1)+r-1}$ from $g_{\mathrm{N+1}}$ and $g_{\mathrm{(N+1)-1}}$.

Let this functional dependence looks like

$$\mathbf{G} = \mathbf{A} \cdot g_N + \mathbf{B} \cdot g_{N-1}, \quad (5)$$

where $\mathbf{A}, \mathbf{B} \in \mathbb{R}^r$ — matrixes in the size $r \times 1$.

We consider the elementary case of functional dependence — the assumption of a constancy g_N during r the sequential intervals of time

$$g_{N+1} = \ldots = g_{N+r-1} = g_N$$

and also a case of linear dependence between $g_N, g_{N+1}, \dots, g_{N+r-1}$

$$g_{N+i-1} = g_N + (i-1) \cdot (g_N - g_{N-1}) = i \cdot g_N + (1-i) \cdot g_{N-1}.$$

Having used (3) and (5) in (4) and having calculated a matrix derivative, we shall find the estimation of intensity g_N

$$g_N = \left((\Phi \cdot A)^T \cdot (\Phi \cdot A) \right)^{-1} \cdot (\Phi \cdot A)^T \cdot \left(C - \mathbf{Q}_{init} - \mathbf{Q}\big|_{g=0} - \Phi \cdot B \cdot g_{N-1} \right) \quad (6)$$

The solution (6) is linear function of the measured concentration c_{ji}, $i = 1, \dots, N + r - 1$ and it is possible to present in the form of the digital filter [5]

$$g_N = \sum_{i=1}^{N+r-1} \sum_{j=1}^{J} f_{j(N-i)} \cdot (c_{ji} - \phi_{ji}^0),$$

where f_{ji} — coefficients of the filter, $f_{j(i-r)} = G_{ji}$, $i = 1, \dots, N + r - 1$

, G_{ji} — solution (6) of the inverse problem at conditions

$$c_{jr} = 1, c_{ji} = 0, i \neq r, Q_{init}(x, y, z, t) = 0.$$

The solution in the form of the digital filter is more effective than other forms in the computing relation because coefficients f_{ji} are calculated once.

Results of computing experiments. Using number of methodical problems numerous quasi-real experiments are lead. Stability numerical approximation to desired value intensity for sources of various types (point, linear, areal, distributed) are constructed, including at presence of measurement errors in sensors ($\delta \in [0; 0,03 \cdot q_{\max}]$). Sensors are settled down outside of an operative range of a source ($f(x,y,z) = 0$) and in an operative range of a source ($f(x,y,z) \neq 0$). The root-mean-square error was used for the account of accuracy of intensity estimation $g(t)$

$$\sigma_G = \sqrt{\frac{1}{N} \cdot \sum_{n=1}^{N} \left(g_n - g(t_n)\right)^2} \,.$$

For each sensor there is the critical step $\Delta t_{st[1]}$, such, that as each step of the solution of the inverse problem $\Delta t > \Delta t_{st[1]}$ the solution is stability, i.e. the step-by-step regularization effect takes place. But its opportunities are limited, since for some sensor $\Delta t_{st[1]}$ can be big enough therefore the estimated solution becomes worse. It is observed that at $r = 1$ with increase Δt dependence of value σ_G from parameter δ weakens.

The desire to increase the accuracy of intensity estimation, reducing a step on time, leads to instability of the solution of inverse problem. sing several sensors ($J > 1$) the sensor with smaller $\Delta t_{st[1]}$ has prevailing influence. In this case it is possible to use function specification method with several ($r > 1$) sequential steps on time.

At $\Delta t = const$ with increase r influence of parameter δ on value σ_G weakens and at certain $r = r_c$ the size σ_G practically does not depend from $\delta \in [0; 0,03 \cdot q_{\max}]$.

The analysis of results of numerical experiments allows to draw a conclusion, that for pair numbers ($\Delta t / \Delta t_{st[1]}$, δ), $\Delta t / \Delta t_{st[1]} \in [0.1;1]$, $\delta \in [0; 0,03 \cdot q_{\max}]$ it is possible to pick up r and in this case errors of estimation $g(t)$ will be minimal.

The information of concentration measurements from sensors is understanding sequentially in the considered method, that allows to organize the on-line monitoring over emissions of pollution in the atmosphere.

References

1. G. I. Marchuk, Mathematical modeling in the problem of environment, Nauka, Moscow, 1982, 320 p.
2. A. N. Tikhonov, V. Ya. Arsenin, Methods of the solution of ill-posed problems, Nauka,

Moscow, 1986, 286 p.

3. O. M. Alifanov, Inverse problems of heat exchange, Mashinostroenie, Moscow, 1988, 280 p.
4. J. Beck, B. Blackwell, Ch. Chent-Clair, Jr., Ill-posed inverse problems of heat conductivity, Mir, Moscow, 1989, 312 p.
5. R. V. Hemming, Digital filters, Sovetskoe radio, Moscow, 1980, 224 p.